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March, 1978 THE USE OF REDUNDANT PACKETS IN SLOTTED-ALOHA-TYPE RANDOM-ACCESS SYSTEMS* ESL-P-809

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ABSTRACT

Slotted-Aloha-type systems are considered in which each transmission is a super-packet consisting of several data packets and some redundant packets in a fixed temporal pattern. Two results are obtained. The first, a negative result, shows that such redundancy, no matter how cleverly exploited, cannot increase the system throughput above that for irredundant transmissions. The second, a positive result, shows that, when the temporal pattern is determined by a simple difference set, average data packet delay can be reduced over an interesting range of throughput values.

I. INTRODUCTION

We consider random-access systems of the "slotted-Aloha-type," by which we mean the following. Each of many information sources sporadically generates binary data in the form of "packets" that are T seconds long (measured in time required for transmission) which it presents to its associated transmitter. If the starting time of the packet lies in the interval $[iT-T, iT)$, then it is transmitted over the common channel in the time "slot" $[iT, iT+T)$, i.e., packets are transmitted in the first available slot after their arrival at the transmitter. When two or more packets are transmitted in the same slot, a "collision" occurs which destroys these packets; otherwise, a transmitted packet is correctly received. Each source eventually learns for each of its transmitted packets, via some feedback mechanism, whether or not that packet was "lost" in a collision. We shall permit "redundancy" in the packets from a given source so that the receiver can sometimes reconstruct a lost packet from other correctly received packets. But, when the receiver cannot reconstruct the lost packet, the source in question must eventually present the lost packet for retransmission.

We now suppose that the sources emit their packets always in the form of superpackets consisting of K information packets and $N-K$ redundant packets (which are completely determined by the data packets) in a fixed temporal pattern. This pattern will be specified by the index vector $(i_1, i_2, \ldots, i_N)$ where $i_1 = 0 < i_2 < \ldots < i_N$, in the manner that if the starting time of the superpacket is t, then the j-th packet in the superpacket starts at time $t + i_j$. In Figure 1,

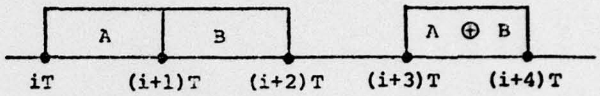

Figure 1 Example of a Superpacket During Transmission with $N=3$, $K=2$ and $(i_1, i_2, i_3) = (0,1,3)$.

we show the structure of a superpacket when $N=3$, $K=2$, the redundant packet is the modulo-two sum of the two information packets, and the temporal pattern has the index vector $(0,1,3)$.

We place a further restriction on the system by requiring that only entire superpackets can be retransmitted, even when only one component packet may have been lost through collisions. We make this restriction to simplify the analysis, but it might also be desirable for the resulting simplification in retransmission procedures.

II. SUPERPACKETS WITH SIMPLE DIFFERENCE SET TEMPORAL PATTERNS

A set of N distinct integers $\{i_1, i_2, \ldots, i_N\}$ is a simple difference set if the $N(N-1)$ differences $i_j - i_k$, $j \neq k$, are all distinct. [A "perfect difference set", sometimes called only a "difference set", is a simple difference set whose differences are also non-zero and distinct when taken modulo $N(N-1)+1$.] The following property shows why the index vector $(i_1, i_2, \ldots, i_N)$ should always be chosen so that its components form a simple difference set.

Property 1: When and only when components of the index vector form a simple difference set, then two colliding transmitted superpackets collide either in all N component packets or in exactly one component packet.

Proof: Suppose first that the components of the index vector do form a simple difference set and that a superpacket starting in slot j collides with a superpacket starting in slot j' in at least two packets. Then, for some integers p, q, r and s, we have

$$
\begin{aligned}
j + i_p &= j' + i_q \\
j + i_r &= j' + i_s
\end{aligned}
\qquad (1)
$$

*This research was supported by the Office of Naval Research under Contract ONR-N00014-64-C-1183.

which implies that $i_p - i_p = i_q - i_s$. But the defining property of a simple difference set then implies that $p = q$ and $r = s$, and hence that $j = j'$ so the superpackets collide in all N packets. Conversely, if the indices do not form a simple difference set, there exist $p \neq q$ such that $i_p - i_r = i_q - i_s$, and hence there exist $j \neq j'$ such that (1) is satisfied. But this implies that two superpackets can collide in more than 1, but less than N, packets. Q.E.D.

The next property should now be obvious, so we omit a formal proof.

Property 2: For a given packet of a given transmitted superpacket, there are exactly $N-1$ starting slots for a transmitted superpacket that collides with the given superpacket in only the given packet. Moreover, these $N-1$ slots are disjoint from the corresponding $N-1$ slots for any other packet of the given superpacket.

III. MAXIMUM DISTANCE SEPARABLE CODES

We shall in the sequel be primarily interested in the case where $K = N-1$ and where the single redundant packet is the modulo-two sum of the $N-1$ information packets. In this case, the receiver can always reconstruct a single lost packet--by subtracting modulo-two each of the other information packets from the redundant packet when an information packet is lost. Thus, a superpacket needs to be retransmitted if and only if two or more of its packets are lost through collisions.

The above simple coding scheme is an example of a "maximum distance separable" [1, pp. 309-311] error-correcting code. If there are m bits in a packet, we can consider a packet to be a single digit in the finite field $GF(2^m)$. The minimum Hamming distance d of a code of length N with K information digits over any field satisfies $d \leq N-K+1$; when equality holds, the code is called maximum distance separable (MDS). The best known MDS codes are the shortened Reed-Solomon codes [1, p. 218] which exist for any $K < N < 2^m$. Thus, the class of MDS codes is rich enough to cover all cases of possible interest in the present application.

It is well-known in coding theory that the maximum number of "erased", or "lost", digits that can be reconstructed in a code word is $d-1$. Thus, when MDS codes are used in the present application, retransmission of a superpacket is necessary if and only if more than $N-K$ packets are lost through collisions.

IV. SYSTEM ANALYSIS

We now proceed to analyze a slotted-Aloha-type multiple-access system with redundant packets in which (1) the components of the index vector form a simple difference set, and (2) the (N,K) code used to determine the redundant packets is maximum distance separable (MDS).

We suppose that the starting times of all the superpackets newly generated by the sources form a stationary Poisson point process whose average number of points in any T second interval is λ_0/K. With this normalization, λ_0 is the average number of information packets per slot (i.e., the information packet rate) and is the appropriate parameter to constrain when comparing systems with different amounts of redundancy. In particular, when the system is in the "equilibrium mode" so that superpackets are being correctly received or reconstructed at the same rate, on the average, as they are being generated by the sources, λ_0 is the system throughput measured in information packets per slot.

Next, we make the Poisson hypothesis (first and boldly made by Abramson [2] for the original "pure Aloha" system) that the starting time of all superpackets presented by the sources for retransmission also form a stationary Poisson point process, independent of the former one, whose average is λ_r points in any T second interval. It is well-known [3] that, although the Poisson hypothesis cannot be strictly justified, it accurately predicts system behavior when the system is operating in its equilibrium mode and when some care has been taken to randomize appropriately the starting times of retransmitted packets.

Because they are the sum of two independent and stationary Poisson point process, the starting times of all superpackets presented for transmission over the common channel form a stationary Poisson process whose average number of points in any T second interval is

$$\lambda = \lambda_0/K + \lambda_r. \tag{2}$$

We shall call λ the channel superpacket rate; and we call $\lambda_p = N\lambda$ the channel packet rate. It follows that the probability p_0 that no superpacket begins in any given channel slot is just

$$p_0 = e^{-\lambda}. \tag{3}$$

Let P_1 denote the probability that a given superpacket is successfully transmitted in the sense that retransmission is unnecessary. Then, when the system is in its equilibrium mode,

$$P_1\lambda = \lambda_0/K$$

since $P_1\lambda$ is then the rate of successfully transmitted superpackets. Equivalently, we can write

$$\lambda_0 = KP_1\lambda = (K/N)P_1\lambda_p. \tag{4}$$

Next, we note that the probability is $e^{-\lambda(N-1)}$ that no superpacket begins in any of the $N-1$ slots which, according to Property 2, would cause a single-packet collision with a given packet of a given transmitted superpacket. Hence, the probability p_I of such an indirect hit on the given packet of the given superpacket is

$$p_I = 1 - e^{-\lambda(N-1)} . \tag{5}$$

Similarly, the probability p_D of a direct hit, i.e., a collision with all N packets, on the given superpacket is

$$p_D = 1 - e^{-\lambda} . \tag{6}$$

Property 2 also implies that indirect hits on distinct given packets of the given superpacket are independent. Thus, the probability of the event A that N - K or fewer indirect hits are made on the given superpacket is

$$P(A) = \sum_{i=0}^{N-K} \binom{N}{i} p_I^{\,i} (1-p_I)^{N-i} . \tag{7}$$

The probability of the event B that no direct hit is made on the given superpacket is

$$P(B) = 1 - p_D = e^{-\lambda} . \tag{8}$$

But, the probability of successful transmission is just

$$P_1 = P(A \cap B) = P(A)P(B) \tag{9}$$

since A and B are independent events.

V. THE SPECIAL CASE OF A SINGLE REDUNDANT PACKET

For the special case where a single redundant packet (equal to the modulo-two sum of the information packets) is used, i.e., for K = N - 1, (7), (8) and (9) yield

$$P_1 = \left[Ne^{-\lambda_P (N-1)^2/N} - (N-1)e^{-\lambda_P (N-1)} \right] e^{-\lambda_P/N} \tag{10}$$

and (4) becomes

$$\lambda_0 = \frac{N-1}{N} P_1 \lambda_P . \tag{11}$$

For this situation, we have plotted the throughput λ_0 versus the channel packet rate $\lambda_P = N\lambda$ for N = 2,3, and 4. For comparison, we have plotted λ_0 also for the ordinary slotted Aloha system without redundant packets, i.e., for K = N = 1, whose describing equations are

$$P_1 = e^{-\lambda_P} \tag{12}$$

$$\lambda_0 = P_1 \lambda_P . \tag{13}$$

The first impression from Figure 2 is one of disappointment. None of the redundant schemes achieves the maximum throughput 1/e = .37 of ordinary slotted Aloha, and the maximum throughput decreases as N increases.

In fact, as we now argue, the use of redundant packets (even without our restrictive assumption that entire superpackets must be retransmitted when packets are lost) can never

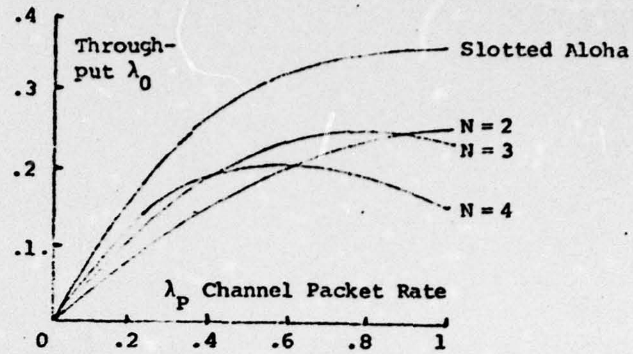

Figure 2 Throughput Comparison of Slotted Aloha with Schemes Using N - 1 Information Packets and 1 Redundant Packet

increase the maximum throughput afforded by slotted Aloha with its selective retransmission of lost packets. For suppose that the N - K redundant packets in a superpacket are used to reconstruct t lost packets. We have already seen that this requires N - K $\geq$ t. But with selective retransmission, we would transmit just the t packets needed to replace those that were lost. Hence, whenever t < N - K, we would avoid some transmissions, thus decreasing the collision probability and increasing the maximum throughput attainable.

We should, however, shun the conclusion that redundant packets have no positive benefits. As we now show, they can decrease the delay with which the information packets are delivered to their destination when the round trip delay (i.e., the time required after transmission to receive an acknowledgement or non-acknowledgement) is large. In this case, the system delay is determined primarily by $1/P_1$, the average number of times that a given packet is transmitted; the larger P_1, the smaller the delay.

In Table I, we give the values of P_1 versus the throughput λ for the same four systems compared in Figure 2. We see from this table that the use of redundancy can indeed increase P_1, for a fixed throughput λ_0, compared to (irredundant) slotted Aloha. A closer comparison shows: (1) that the N = 2 system is never superior to slotted Aloha, (2) that the N = 3 system is superior to slotted Aloha for $0 < \lambda_0 < .185$, and (3) that the N = 4 system is superior to slotted Aloha for $0 < \lambda_0 < .125$ and to the N = 3 system for $0 < \lambda_0 < .064$. The improvements afforded by redundant packets are more substantial than the closeness of the numbers in Table I might seem to indicate--for instance, at $\lambda_0 = .10$, the N = 3 system uses about 25% fewer retransmissions than slotted Aloha since the retransmission probabilities $(1 - P_1)$ are .085 and .106, respectively.

Throughput λ_0	P_1 (N=2 system)	(N=3 system)	(N=4 system)	(Slotted Aloha)
.05	.946	.965	.969	.949
.10	.884	.915	.909	.894
.15	.804	.849	.813	.836
.20	.710	.755	.633	.771

Table I Comparison of the Probability of Successful Transmission for Slotted Aloha and Schemes Using N-1 Information Packets and 1 Redundant Packet

VI. CONCLUSIONS

We have shown that the use of redundant packets in a random-access situation, while it cannot increase system throughput over that afforded by the use of irredundant packets with selective retransmission, can provide improved delay characteristics.

We should emphasize that we demonstrated the improved delay characteristics using only the simplest possible MDS codes, viz., those with a single redundant packet. It seems to us that the potential of more powerful MDS codes ought certainly to be explored. Moreover, we also imposed the stringent requirement (mostly for convenience in our analysis) that entire superpackets must always be transmitted. The potential further gain when this restriction is removed ought also to be investigated.

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